

Permeabilities of metamaterials

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Abstract

Scattering of electromagnetic (EM) waves by many small particles, embedded in a given medium, is studied. Physical properties of the particles are described by their boundary impedances. The limiting equation is obtained for the effective EM field in the limiting medium, in the limit $a \rightarrow 0$, where a is the characteristic size of a particle and the number $M(a)$ of the particles tends to infinity at a suitable rate. An analytical formula for the permeability $\mu(x)$ of the limiting medium is given. Analysis of this formula allows one to find out the range of the values of the permeability in the material, obtained by embedding many small particles.

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1 Introduction

In this paper we outline a theory of monochromatic electromagnetic (EM) wave scattering by many small particles (bodies) embedded in a homogeneous medium which is described by the permittivity $\epsilon' = \epsilon_0 + i\frac{\sigma}{\omega}$, $\epsilon_0 > 0$, and constant permeability μ_0 . The wave number $k = \omega(\epsilon'\mu_0)^{1/2}$. Related papers are [4]-[8], and the results we use without proof are taken mostly from [4]. The small particles are embedded in a finite domain Ω . Smallness of the particles means that $|k|a \ll 1$, where a is the characteristic size of the particles, and k is the wave number. The medium, created by the embedding of the small particles, has new physical properties. In particular, it has a spatially inhomogeneous magnetic permeability $\mu(x)$, which can be

controlled by the choice of the boundary impedances of the embedded small particles and their distribution density. This is a new physical effect. An analytic formula for the permeability of the new medium is derived in [4]:

$$\mu(x) = \frac{\mu_0}{\Psi(x)}, \quad (1)$$

where

$$\Psi(x) = 1 + \frac{8\pi i}{3\mu_0\omega} h(x)N(x). \quad (2)$$

Here ω is the frequency of the EM field, ϵ_0 is the constant dielectric parameter of the original medium, $h(x)$ is a function describing boundary impedances of the small embedded particles, and $N(x) \geq 0$ is a function describing the distribution of these particles. We assume that in any sub-domain Δ , the number $\mathcal{N}(\Delta)$ of the embedded particles D_m is given by the formula:

$$\mathcal{N}(\Delta) = \frac{1}{a^{2-\kappa}} \int_{\Delta} N(x) dx [1 + o(1)], \quad a \rightarrow 0, \quad (3)$$

where $N(x) \geq 0$ is a continuous function, vanishing outside of the finite domain Ω in which small particles (bodies) D_m are distributed, $1 \leq m \leq M$, $M = M(a)$, $\kappa \in (0, 1)$ is a parameter, and the boundary impedances of the small particles are defined by the formula

$$\zeta_m = \frac{h(x_m)}{a^\kappa}, \quad x_m \in D_m, \quad (4)$$

where x_m is a point inside m -th particle D_m , $\text{Re } h(x) \geq 0$, and $h(x)$ is a continuous function vanishing outside Ω . The impedance boundary condition on the surface S_m of the m -th particle D_m is $E^t = \zeta_m[H^t, N]$, where E^t (H^t) is the tangential component of E (H) on S_m , and N is the unit normal to S_m , pointing out of D_m . Physical properties of the impedance ζ are discussed in [2]. In particular, $\text{Re } \zeta \geq 0$.

Since one can choose the functions $N(x)$ and $h(x)$, one can create a desired magnetic permeability in Ω . This is a novel idea, to the author's knowledge.

We have also derived in [4] an analytic formula for the refraction coefficient of the medium in Ω created by the embedding of many small particles.

$$K^2(x) = \frac{k^2}{1 + \frac{8\pi i}{3\mu_0\omega} h(x)N(x)}, \quad k^2 = \omega^2 \epsilon' \mu_0, \quad (5)$$

where the coefficient $\frac{16\pi}{3}$ appears if D_m are balls of radius a centered at the points x_m . For the small bodies D_m of arbitrary shape the coefficient $\frac{8\pi}{3}$

should be replaced by a tensorial coefficient $c_m = O(1)$, depending of the shape of D_m .

An equation for the EM field in the limiting medium is derived in [4]:

$$E(x) = E_0(x) - \frac{8\pi i}{3\mu_0\omega} \nabla \times \int_{\Omega} g(x, y) h(y) N(y) \nabla \times E(y) dy. \quad (6)$$

The limiting medium is created when the size a of small particles tends to zero while the total number $M = M(a)$ of the particles tends to infinity at the rate determined by (3).

The refraction coefficient in the limiting medium is spatially inhomogeneous.

Our theory may be viewed as a "homogenization theory", but it differs from the usual homogenization theory (see, e.g., [1], [3], and references therein) in several respects: we do not assume any periodic structure in the distribution of small bodies, our operators are non-selfadjoint, the spectrum of these operators is not discrete, etc. Our ideas, methods, and techniques are quite different from the usual methods.

These ideas are similar to the ideas developed in papers [5, 6], where scalar wave scattering by small bodies was studied, and in the papers [4] and [8], where EM wave scattering by many small particles was studied. Scattering of EM waves brought new technical difficulties which were resolved in [4] and [8]. The difficulties come from the vectorial nature of the boundary conditions. Results from [7] were used for a justification of the passages to the limit $a \rightarrow 0$.

In [4] a new numerical method for solving many-body wave-scattering problems for small scatterers is given.

We predict theoretically the new physical phenomenon: by embedding many small particles with suitable boundary impedances into a given homogeneous medium, one can create a medium with a desired spatially inhomogeneous permeability (1).

One can create material with a desired permeability $\mu(x)$ by embedding small particles with suitably chosen boundary impedances. Indeed, by formula (1) one can choose a complex-valued, in general, function $h(x)$, and a non-negative function $N(x) \geq 0$, describing the density distribution of the small particles, so that the right-hand side of formula (1) will yield a desired function $\mu(x)$.

The goal of this paper is to discuss the possible values of magnetic permeability one can get in the metamaterial, obtained by embedding many small impedance particles in a given material.

This discussion is given in the next Section.

2 Possible values of permeability

Let us denote $q = q(x) := \frac{8\pi N(x)}{3\omega\mu_0}$, and rewrite formula (1) as

$$\mu(x) = \frac{\mu_0(1 - qh_2 - iqh_1)}{(1 - qh_2)^2 + q^2h_1^2} \quad (7)$$

Quantity $q(x) > 0$. Thus, the values of $\mu(x)$ can be controlled by choosing the function h in equation (4).

Conclusion:

Assume that the impedances of the small particles are defined by formula (4), where $h(x) = h_1(x) + ih_2(x)$ is a function with $\text{Re}h(x) = h_1(x) \geq 0$. Let $\mu(x) = \mu_1(x) + i\mu_2(x)$, where $\mu_1(x)$ is the real part of $\mu(x)$. Then it is not possible to create metamaterial with $\mu_2(x) > 0$ by embedding many small impedance particles into a given material. Indeed,

$$\mu_2(x) = -\frac{\mu_0qh_1(x)}{(1 - qh_2(x))^2 + q^2h_1^2(x)} < 0,$$

provided that μ_0, q and h_1 are all positive, which is the case. The assumption $h_1(x) \geq 0$ is a necessary assumption for boundary impedances, see [2], p.301.

For high frequencies ω the quantity $q(x) = O(\frac{1}{\omega})$ is small, so if $h_1(x) = O(1)$, then $\mu_2(x) = O(\frac{1}{\omega})$ is also small.

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